On Informative ness of Variation Coefficients While Analysis Signals Structural Properties

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Abstract— For effective analysis of optical signals it is important to obtain the parameters of their structural properties, variation coefficient, excess, asymmetric coefficient can be used for this purposes. The use of variation coefficient is being possible when at comparatives analysis of structurally similar signals. That in why the variation coefficient would be implied at structural analysis of optical signals in the same intervals found. There are properties of factors variations of distributions probabilities critically investigated for their use in applied problems of the statistical analysis. Their greatest information is shown for structural properties of distributions of modules of a deviation from an average values of investigated laws of distributions.

Keywords— Processing of one-dimensional and many-dimensional signals.

Introduction

Variation coefficient, defined as degree of dissipation of random variable at its mean value [1], is among many numerical characteristics of probabilities distributions (statistic data), while are used in applied problems of statistic investigations

$$V_{\hat{y}} = \sqrt{\mu_{2y}} / \alpha_{1y} \Rightarrow \delta_y / \alpha_{1y}$$

where: α_{1y} and μ_{2y} are respectively the first initial (mean value) and second central (dispersion) moments of random variable; δ_y - mean-square deviation of random variable.

Frame the definition of variation coefficient it follows that $V_{\hat{y}}$ is a dimensionless value. This coefficient was proposed by K. Pirson. Quid here it was supposed that variation coefficient should be used at positive values of α_{1y}. The last limitation is not quite clear. Proceeding brome the physics of variation coefficient, characterizing distribution structure, it is advisable to present it in the form

$$V_{\hat{y}} = \delta_y / |\alpha_{1y}|$$

Let us consider variation coefficient for some parametric distributions:

- Uniform distribution $f_{\hat{y}}(y) = 1/(y^+ - y^-)\ y \in [y^-, y^+]$ initial moment - α_{1p} nd mean – square deviation for being considered distribution are respectively equal:

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$$\alpha_{1\eth} = (y^{+} + y^{-})/2$$

Taking into account last relations, variation considered for uniform distribution will be of the following form

$$V_p = \sqrt{\frac{1}{3}\left(1 - \frac{y^{+}y^{-}}{\alpha_{1p}^{2}}\right)}$$

That is V_p depends both on interval length $[y^{-}, y^{+}]$ and on its position (parameters).

• Exponential distribution $f_{\acute{y}}(y) = \lambda e^{\lambda y}$, $y \in [0, \infty]$, $\lambda > 0$

The first initial moment and mean – square deviation of exponential distribution are respectively equal

$$\alpha_{1\acute{Y}} = 1/\lambda, \quad \sqrt{\mu_{2\acute{Y}}} = \delta_{\acute{Y}} = 1/\lambda$$

then variation coefficient will be equal distribution

$$V_{\acute{Y}} = (1/\lambda)/(1/\lambda) = 1$$

that is, it does not depend on parameter and is constant.

• Normal distribution

$$f_{\acute{y}}(y) = \frac{1}{\sqrt{2\pi}\delta_N} \cdot \exp\left(-\frac{(y - \alpha_N}{2\delta_N^2}\right) \quad y \in (-\infty, \infty)$$

where δ_N and α_{1N} - are parameter which coincide respectively with initial moment and mean – square deviation of normal distribution. Variation coefficient for normal distribution has the form

$$V_N = \delta_N / \alpha_{1N}$$

that is it is impossible to identify any of its parameter and its structure according to variation coefficient of normal distribution.

• Exponential distribution $f_{\acute{y}}(y) = \tilde{n} y^{\tilde{n}-1}$, $\acute{o} \in [0,1]$, $c > 0$.

The first initial moment and mean – square deviation of exponential distribution are deviations of exponential distribution are determined respectively according to formulae:

$$\alpha_{1\tilde{n}} = \int_0^1 \acute{o}\tilde{n}\acute{o}^{\tilde{n}-1} dy = \frac{c}{c+1};$$

$$\delta_c = \sqrt{\mu_{2c}} = \left(\int \acute{o}^2 \tilde{n}\acute{o}^{\tilde{n}-1} dy - \alpha_{1c}^2\right)^{\frac{1}{2}} = \left(\frac{c}{c+2} - \alpha_{1c}^2\right)^{\frac{1}{2}} = \sqrt{\frac{(1-\alpha_{1c})^2 \alpha_{1c}}{(2-\alpha_{1c})}}$$

Taking into account the following sequence of transformations

$$\left(\alpha_{1\tilde{n}} = \frac{c}{c+1}\right) \Rightarrow (c\alpha_{1\tilde{n}} + \alpha_{1\tilde{n}} = c) \Rightarrow \left(c = \frac{\alpha_{1\tilde{n}}}{1 - \alpha_{1\tilde{n}}}\right)$$

it follows, that

$$V_c = \sqrt{\frac{(1-\alpha_{1c})^2}{\alpha_{1c}(2-\alpha_{1c})}},$$

Accepted variation coefficient depends only on the first initial moment.

Bernoulli distribution. Let us consider the following general case. Let values $\acute{o}^{+}$ and $\acute{o}^{-}$ be given with outcome probabilities in the experiment $\text{Ð}(\acute{o}^{+}) = \text{Ð}$ and $\text{Ð}(\acute{o}^{-}) = 1 - \text{Ð}$. Then the first initial and the second central moments will be respective by equal

$$\alpha_{1\acute{A}} = \acute{o}^{+} \cdot \eth + \acute{o}^{-} \cdot (1 - \eth) = (\acute{o}^{+} - \acute{o}^{-}) \cdot \eth + \acute{o}^{-},$$

$$\alpha_{2\acute{A}} = (\acute{o}^{+} - \acute{o}^{+}\eth) + (\acute{o}^{-} - \acute{o}^{+}\eth)^2 = (\acute{o}^{+} - \acute{o}^{-})^2 \eth(1 - \eth)$$

Taking into account accepted expressions for variation coefficient it will by true that

$$V_{\acute{A}\acute{o}} = \sqrt{\frac{(\acute{o}^{+} - \acute{o}^{-})^2 \eth(1-\eth)}{((\acute{o}^{+} - \acute{o}^{-})\eth + \acute{o}^{-})^2}}$$

which depends on the parameter P, interval - $(\acute{o}^{+} - \acute{o}^{-})$ and position - $\acute{o}^{-}$ possible values of random variables.

Basic relations, connected with variation coefficient for considered above parametric distributions of probabilities are given in Table 1.

Distributions	Indices		
	$\alpha_{1(\cdot)}$	$\delta_{1(\cdot)}$	$V_{1(\cdot)}$
Uniform	$(y^{+} + y^{-})/2$	$\left(\frac{\alpha_{1p}^2 - y^{+} \cdot y^{-}}{3}\right)^{\frac{1}{2}}$	$\left(\frac{1}{3}\left(1 - \frac{y^{+} \cdot y^{-}}{\alpha_{1p}^2}\right)\right)^{\frac{1}{2}}$
Exponential	$1/\lambda$	$1/\lambda$	1
Normal	α_{1N}	δ_N	δ_N / α_{1N}
Power	$\frac{c}{c+1}$	$\left(\frac{(1-\alpha_{1\tilde{n}})^2 \cdot \alpha_{1\tilde{n}}}{2 - \alpha_{1\tilde{n}}}\right)^{\frac{1}{2}}$	$\left(\frac{(1-\alpha_{1\tilde{n}})^2}{\alpha_{1\tilde{n}}(2-\alpha_{1\tilde{n}})}\right)^{\frac{1}{2}}$
Bernoully	$(\acute{o}^{+} - \acute{o}^{-})\eth + \acute{o}^{-}$	$(\acute{o}^{+} - \acute{o}^{-})^2 \sqrt{\eth(1-p)}$	$\left(\frac{(\acute{o}^{+} - \acute{o}^{-})^2 \eth(1-\eth)}{((\acute{o}^{+} - \acute{o}^{-})\eth + y^{-})^2}\right)^{\frac{1}{2}}$

Table 1. Table of indices of variation coefficient for some parametric distributions

Let us consider distributions of general form. Let density of distributions $[y^{+}, y^{-}]$ by given in the interval $f_{\acute{y}}(y)$, $\acute{o} \in [y^{-}, y^{+}]$. Linear transformation of random variable

$$x = ay + b, \; x \in [(ay^{-} + b), (ay^{+} + b)], \; y \in [y^{-}, y^{+}]$$

does not change distributions structure (class of distribution). If to admit

$$x^{-} = ay^{-} + b = 0 \text{ и } x^{+} = ay^{+} + b = 1$$

that the following expression will by true for x

$$x = (y - y^{-})/(y^{+} - y^{-}), \; y \in [y^{-}, y^{+}], x \in [0,1]$$

Initial (the first) and central moments of distribution $f_{\hat{x}}(x)$ meet the following inequality [1]

$$0 \le \lambda_{1x} \le 1,$$
$$0 \le \mu_{kx} \le p(1-p)[(1-p)^{k-1} + (-1)p^{k-1}] = \mu_{k\acute{A}}$$

where α_{1x} and μ_{1x} - the first initial and central moments of k-order, respectively; $\mu_{k\acute{A}}$ - central moments of k-order of Bernoulli distribution

$$P(x=1) = p = \alpha_{1x}, \quad P(x=0) = (1-p) = (1-\alpha_{1x})$$
$$\frac{\sqrt{\mu_{2\acute{o}}}}{\alpha_{1\acute{o}}} = \frac{(\acute{o}^{+} - \acute{o}^{-})\sqrt{\mu_{2\eth}}}{\alpha_{1\eth}(\acute{o}^{+} - \acute{o}^{-}) + \acute{o}^{-}}; \quad \frac{\sqrt{\mu_{2x}}}{\alpha_{1x}} = \frac{\sqrt{\mu_{2\acute{o}}}}{\alpha_{1\acute{o}} - \acute{o}^{-}}$$

Basic indices of variation coefficients for distributions reduced to the interval [0, 1], given in Table 1, are given in Table 2.

Distributions	Indices		
	$\alpha_{1\eth(\cdot)}$	$\delta_{(\cdot)}$	$V_{\eth(\cdot)}$
Uniform	0,5	0,283	0,56
Exponential			
Normal	0,5		
Power	$\frac{c}{c+1}$	$\frac{(1-\alpha_{1\tilde{n}})^2 \cdot \alpha_{1\tilde{n}}}{2-\alpha_{1\tilde{n}}}$	$\left(\frac{(1-\alpha_{1\tilde{n}})^2}{\alpha_{1\tilde{n}}(2-\alpha_{1\tilde{n}})}\right)^{\frac{1}{2}}$
Bernoulli	$\eth$	$\sqrt{\eth(1-\eth)}$	$\sqrt{(1-\eth)/\eth}$

Table 2.Table of indices of variation coefficients for distributions reduced to the interval [0, 1].

Indices $\alpha_{1\eth(\cdot)}$, $\delta_{(\cdot)}$ presented in the Table 2 and dependences $V_{\eth(\cdot)}$ are given respectively in Fig.1.

Analysis of graphs, given in Fig.2, shows that variation coefficients are most sensitive in the field of low bound of the interval [0, 1]. This property speaks on the efficiency of using variation coefficient of deviation module distributions from mean value of investigated distribution.

That, informative ness of variation coefficients to characterize structural properties of distributions is high only at the analysis of distributions, defined in the same interval.

The most useful is consideration of distributions, reduced to the interval [0, 1]. Variation coefficients are better adapted to the value of distributions of deviation module from their mean value.

Variation coefficients of distributions reduced to the interval [0, 1] are limited at the low bound by «0» and at upper bound by the values of variation coefficient of Bernoulli distribution with the parameter, equal to the first initial moments of investigated distributions.

Thus, for effective analysis of optical signals it is important to obtain the parameters of their structural properties, variation coefficient, excess, asymmetric coefficient can be used. The use of variation coefficient is being possible when at comparatives analysis of structurally similar signals.

That in why the variation coefficient would be implied at structural analysis of optical signals in the same intervals found. There are properties of factors variations of distributions probabilities critically investigated for their use in applied problems of the statistical analysis.

Their greatest information is shown for structural properties of distributions of modules of a deviation from an average values of investigated laws of distributions.

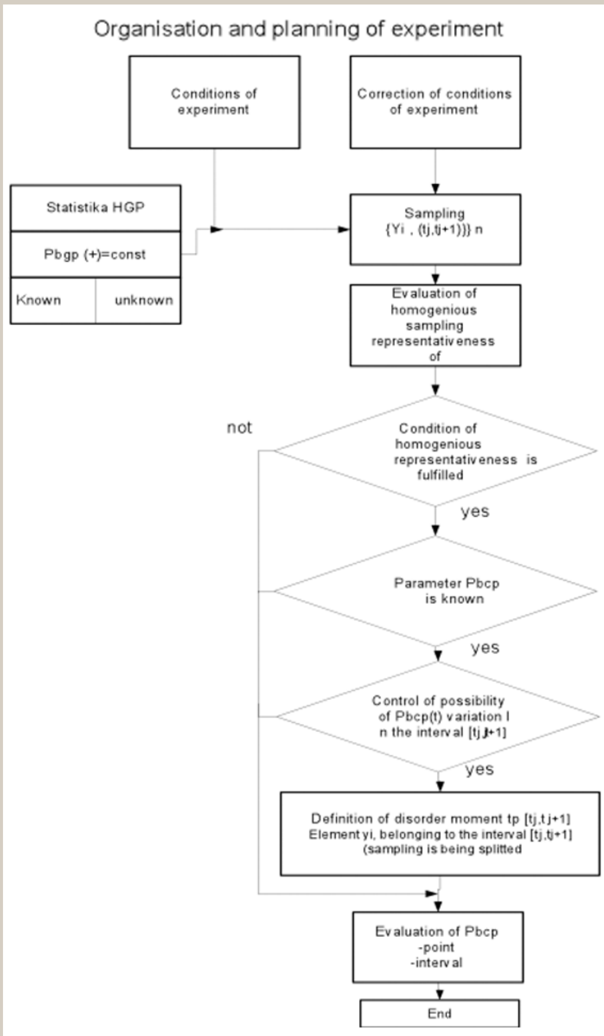

Figure 1. Generalized scheme of the analysis of static homogenous general population.

Conclusions

1. Informative ness of variation coefficients to characterize structural properties of distributions is high only at the analysis of distributions, defined in the same interval. The most useful is consideration of distributions, reduced to the interval [0, 1].

2. Variation coefficients are better adapted to the value of distributions of deviation module from their mean value.

3. Variation coefficients of distributions reduced to the interval [0, 1] are limited at the low bound by «0» and at upper bound by the values of variation coefficient of Bernoulli distribution with the parameter, equal to the first initial moments of investigated distributions.

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Tatiana I. Lapina is interesrted in aspects of the creation of a system for objects monitoring based on informational statistical approach, processing of one-dimensional and many-dimensional signals.